In the Name of GOD

Energy of bandpass signal based on the Energy of equ. baseband signal

$$\underbrace{s(t)}_{\text{bandpass signal}} = Re\left\{ \underbrace{s_e(t)}_{\text{equ. baseband (low pass) signals}} e^{j2\pi f_c t} \right\}$$

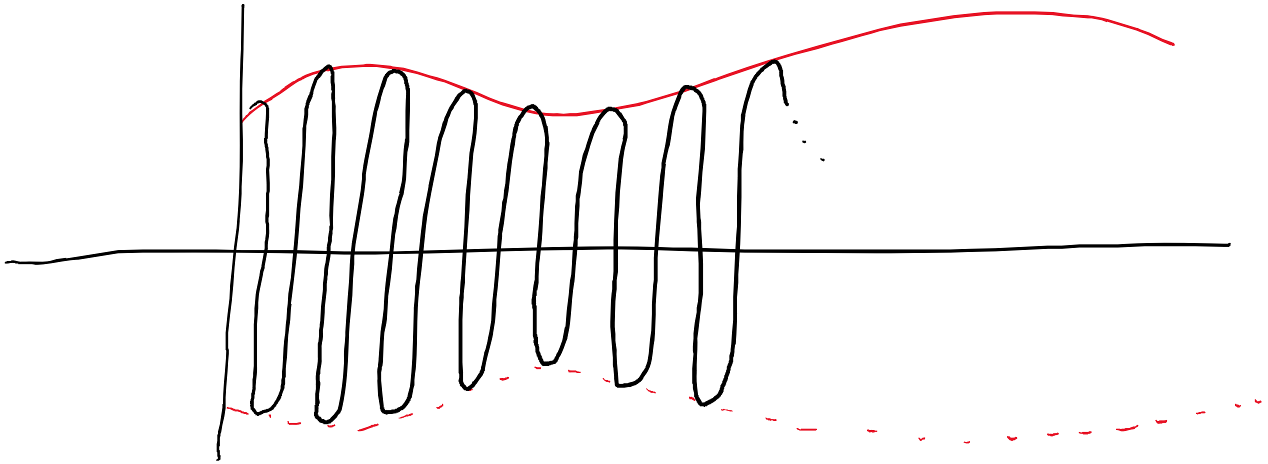
$$\underbrace{E_s}_{\text{Energy of } s(t)} = \frac{1}{2} \underbrace{E_{s_e}}_{\text{Energy of } s_e(t)}$$

$$\int a^2(t)\cos(4\pi f_c t + 2\theta(t))\,dt = 0$$

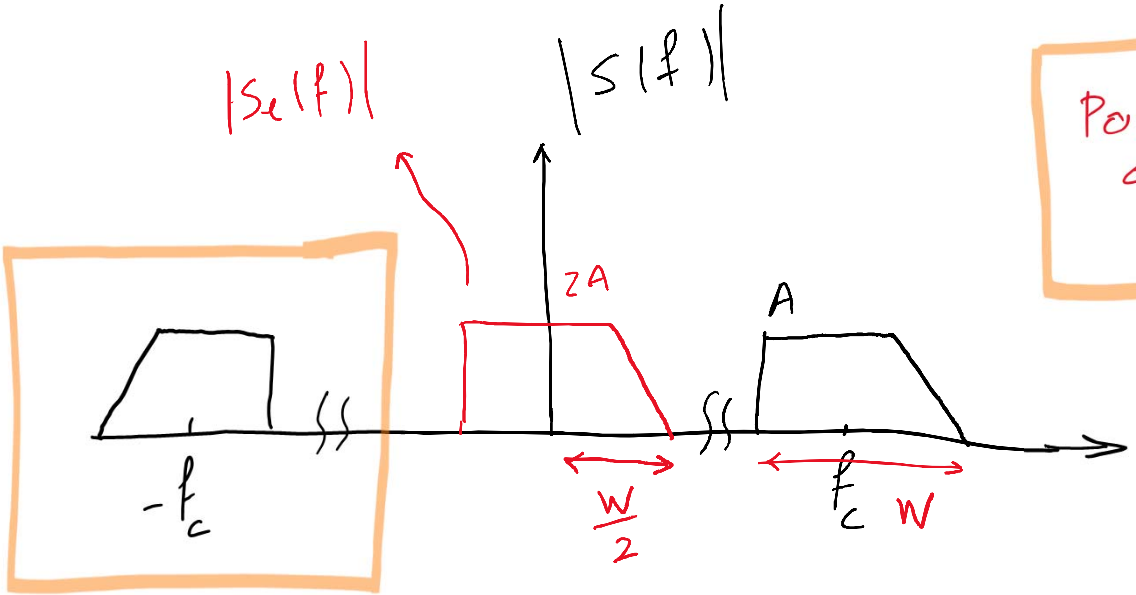
because $W \ll f_c$ that means time variation of $a(t)$ is so smaller than that of $c(4\pi f_c t + 2\theta(t))$. So the above integration is equal to zero.

$a^2(t)$ :

$a(t)$ : envelope of $s_e(t)$



$a(t)$ is approximately constant in comparison to $\cos(4\pi f_c t + 2\theta(t))$




$$\underbrace{P_s}_{\text{Power of } s(t)} = \underbrace{P_{s_e}}_{\text{Power of } s_e(t)}$$

$$\mathcal{E}_s = \frac{1}{2}\mathcal{E}_{s_e}$$

BW of $s_e(t)$ is $\frac{1}{2}$ of that of $s(t)$

Now, we want to find an expression for spectrum of bandpass signal $s(t)$ based on the spectrum of equ. baseband signal. As you know, we have

$$s(t) = \mathrm{Re}\left\{ s_\ell(t)\, e^{j2\pi f_c t} \right\}$$

$$\xrightarrow{FT\{\cdot\}} \quad FT\{s(t)\} = FT\left\{ \mathrm{Re}\left\{ s_\ell(t)\, e^{j2\pi f_c t} \right\} \right\}$$

$$\Longrightarrow \quad S(f) = FT\left\{ \mathrm{Re}\left\{ s_\ell(t)\, e^{j2\pi f_c t} \right\} \right\}$$

we know for any Complex parameter $Z$, we have

$$\mathrm{Re}\{z\} = \frac{1}{2}\{Z + Z^*\}$$

$$\Rightarrow \quad s(f) = FT\{\underbrace{\mathrm{Re}\{S_e(t)\, e^{j2\pi f_c t}\}}_{\frac{1}{2}\{S_e(t)\, e^{j2\pi f_c t} + S_e^*(t)\, e^{-j2\pi f_c t}\}}\}$$

$$\Rightarrow s(f) = \frac{1}{2} FT\{S_e(t)\, e^{j2\pi f_c t} + S_e^*(t)\, e^{-j2\pi f_c t}\}$$

$$s(f) = \frac{1}{2}[S_e(f - f_c) + S_e^*(-f - f_c)]$$

FT characteristics

$$\Rightarrow \quad S(f) = \frac{1}{2}\left[ S_\ell (f - f_c) + \underline{S_\ell^* (-f - f_c)} \right]$$

1) $s(t) = \mathrm{Re}\left\{ s_\ell(t)\, e^{j 2\pi f_c t} \right\}$

2) $s(t) = x(t) \cos 2\pi f_c t - y(t) \sin 2\pi f_c t$

3) $s(t) = a(t) \cos\left(2\pi f_c t + \theta(t)\right)$

$s_\ell(t) = x(t) + j\, y(t)$ , $a(t) = \sqrt{x^2(t) + y^2(t)}$ , $\theta(t) = \tan^{-1} \dfrac{y(t)}{x(t)}$

# Simulation Assignment # 1

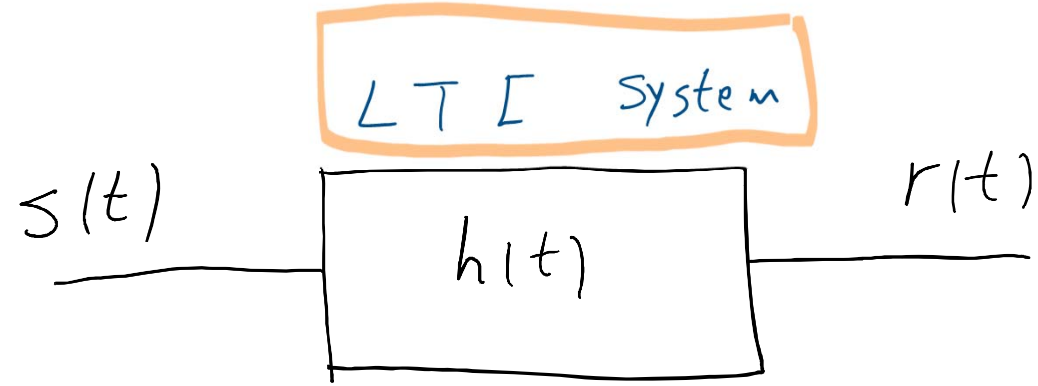
We want to write a code in python (or MATLAB) to plot the power spectrum of a signal s(t).

In the first step, suppose that s(t) is a deterministic signal.

And in the second step modify your code so that it can plot the PSD of an stochastic signal s(t).

* How to represent a bandpass system, based on equ. baseband system

In this part we want to talk about Linear Time Invariant bandpass systems and their equ. baseband systems.

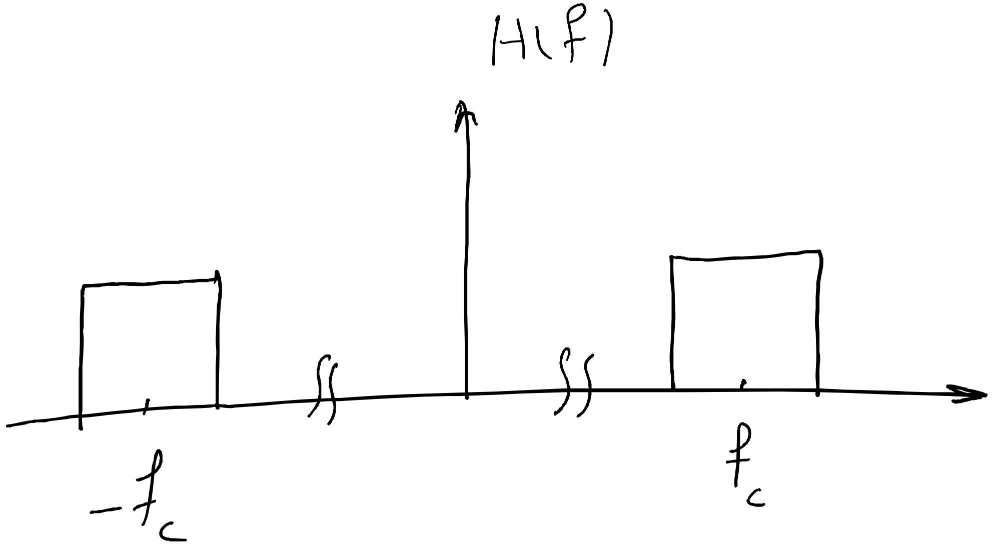



$$r(t) = s(t) * h(t)$$

$$R(f) = S(f)\,H(f) \quad (*)$$

The output of any band-pass is obviously a band-pass signal. So $r(t)$ is a bandpass signal.

As you know any LTI system is known by its Impulse Response $h(t)$




$$\underbrace{H(f)}_{\uparrow} = FT\{h(t)\}$$

Transfer function of the system

Also, in this part we talk about band-pass systems that their transfer functions is located in high frequencies and have Hermitian symmetry. (because Impulse Response h(t) is a real function)

So, we want to find an expression for $r(t)$ based on equ. lowpass signal and the equ. lowpass system.

We start from the equ. low-pass system.

$$H_\ell(f-f_c) = \begin{cases} H(f) & f>0 \\ 0 & f<0 \end{cases} \quad (1)$$

Hermitian Symmetry $\Longrightarrow$

$H^*(-f) = H(f)$ (*)

$$H_\ell^*(-f-f_c) = \begin{cases} 0 & f>0 \\ \underbrace{H^*(-f)}_{H(f)} & f<0 \end{cases} \quad (2)$$

$$①, ② \Longrightarrow H(f) = H_\ell(f-f_c) + H_\ell^*(-f-f_c)$$

$$\xrightarrow{FT^{-1}} h(t) = 2\,Re\{h_\ell(t)\, e^{j2\pi f_c t}\}$$

Now, we want to find an expression for $r(t)$ and its equ. lowpass signal and equ. lowpass system.

$$\left.\begin{aligned} r(t) &= s(t) * h(t) \\ R(f) &= S(f)\, H(f) \end{aligned}\right\} \text{all are band-pass signals and system}$$

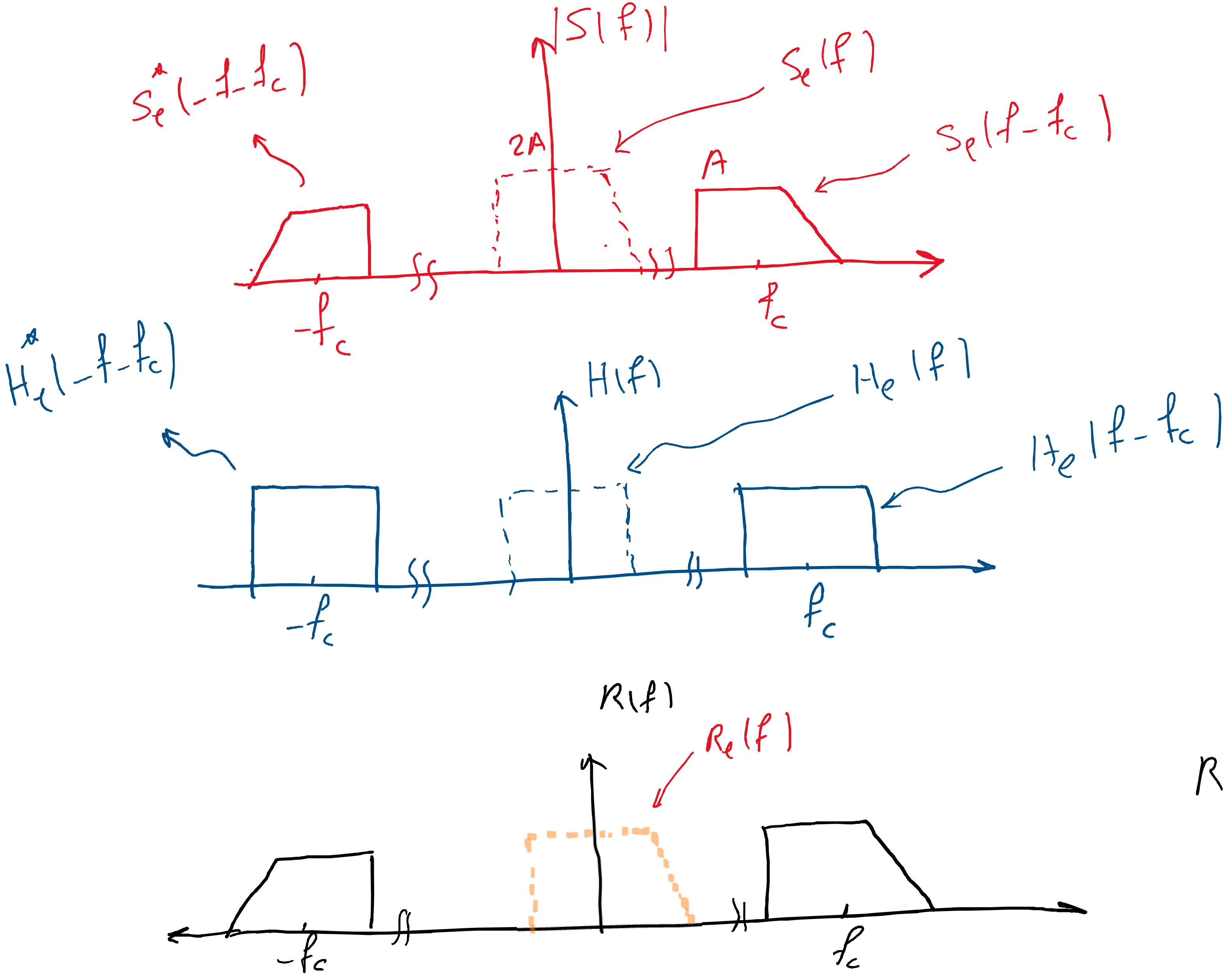
$$R(f) = S(f)\,H(f)$$

$$= \frac{1}{2}\left[S_\ell(f-f_c) + S_\ell^*(-f-f_c)\right]\left[H_\ell(f-f_c) + H_\ell^*(-f-f_c)\right]$$

$$= \frac{1}{2}\left[S_\ell(f-f_c)\,H_\ell(f-f_c) + S_\ell^*(-f-f_c)\,H_\ell^*(-f-f_c)\right]$$

$$+ \frac{1}{2}\left[\underbrace{S_\ell(f-f_c)\,H_\ell^*(-f-f_c)}_{0} + \underbrace{S_\ell^*(-f-f_c)\,H_\ell(f-f_c)}_{0}\right]$$

$$\Longrightarrow R(f) = \frac{1}{2}\left[S_\ell(f-f_c)\,H_\ell(f-f_c) + S_\ell^*(-f-f_c)\,H_\ell^*(-f-f_c)\right]$$



|S(f)|
S_e^*(-f-f_c)
S_e(f)
S_e(f-f_c)
2A
A
-f_c
f_c
H(f)
H_e^*(-f-f_c)
H_e(f)
H_e(f-f_c)
-f_c
f_c
R(f)
R_e(f)
-f_c
f_c


$$R(f) = S(f)\,H(f)$$

$$R(f) = \frac{1}{2}\left[ S_\ell(f - f_c) H_\ell(f - f_c) + S_\ell^*(-f - f_c) H_\ell^*(-f - f_c) \right]$$

$$\boxed{R_\ell(f) = S_\ell(f) H_\ell(f)} \Longrightarrow R(f) = \frac{1}{2}\left[ R_\ell(f - f_c) + R_\ell^*(-f - f_c) \right]$$

$$\Rightarrow r(t) = Re\left\{ r_\ell(t)\, e^{j2\pi f_c t} \right\}$$

$$r_\ell(t) = s_\ell(t) * h_\ell(t)$$

So, we Can Conclude that the Performance of a band-pass Commu. System, is equal to that of baseband equivalent System.

So, we Can do our mathematical analysis of a band-pass system, in the equ. base-band form. Because the Math. analysis in baseband freq. is so Simpler than that of in band-pass freq. We know that no information will be missed by this operation